## THE GFT TEMPLATES: WHY AND HOW TO USE THEM

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### The General Feedback Theorem (GFT) identifies the correct loop gain even in the presence of inevitable nonidealities. The Intusoft ICAP/4 GFT Templates control all the simulations and identify separately the nonidealities.

Feedback systems are usually designed with the familiar single-loop block diagram in mind, that is, the major loop feedback path is designed to have a transmission function such that the specified gain is achieved in the presence of noninfinite major loop gain. Various *nonidealities* make the single-loop block diagram progressively less useful, especially at higher frequencies.

Nonidealities invariably exist in real electronic circuits, and include direct forward transmission through the feedback path, reverse transmission through the forward path, and nonzero reverse loop gain.

The General Feedback Theorem (GFT) takes into account the nonidealities and discloses whether or not they have a significant effect upon the result.



#### Fig. 1. Conventional block diagram.

In conventional analysis, the single-loop block diagram of Fig. 1 is assumed and the *closed-loop gain H* is given by

$$H = \frac{A}{1 + AK} = H_{\infty} \frac{T}{1 + T} = H_{\infty} \frac{1}{1 + \frac{1}{T}} = H_{\infty} D$$
(1)

where T = AK is the *loop gain* and  $H_{\infty}$  is the *ideal closed-loop gain*, so called because it is the closed-loop gain that would be achieved if the loop gain T were infinite. The *discrepancy factor* D = 1/(1+1/T) is a unique function of T, and shows how Hdeviates from  $H_{\infty}$  as T crosses over 0dB, and acquires a significant peak if the phase margin of T is below about 52. From a design point of view, the last version of H in (1) is more useful than the first version, because  $H_{\infty} = 1/K$ , the reciprocal of the feedback ratio K, and is the known design specification, so that the design problem reduces to ensuring that D is close enough to unity over the required frequency range.

Attention is usually focused on the loop gain T, which is calculated by injection of a test signal into the forward path as in Fig. 2. The u's are either all voltages or all currents.



# Fig. 2. Conventional method for calculation of loop gain T, by injection of a single test signal.

There are two deficiencies in the conventional approach:

- 1. The right answer for T is obtained only if the test signal is injected at an "ideal point," such as at an ideal dependent generator.
- 2. Nonidealities are ignored, as is tacitly conceded in representation of the forward and feedback signal paths by arrows to exclude reverse transmission.

The General Feedback Theorem (GFT) overcomes both deficiencies, and provides additional benefits.

- 1. The loop gain T is calculated by injection of both voltage and current test signals at an injection point that may or may not be ideal.
- 2. Calculations are performed on the complete circuit model without the constraint of unidirectional paths.
- 3. No assumptions or approximations are made, so the nonidealities are included. If the voltage and current test signal injections are made at the error signal summing point, effects of the nonidealities can be evaluated separately.
- 4. The GFT dissects the closed-loop gain into three constituent transfer functions, each of which is obtained directly in terms of the circuit elements, which makes it possible for the results to be used backwards for design. This is the principal objective of DESIGN-ORIENTED ANALYSIS, The Only Kind of Analysis Worth Doing.

The test signal injection configuration is shown in Fig. 3, and the GFT discloses that an additional term must be included in (1):

$$H = H_{\infty} \frac{1 + \frac{1}{T_n}}{1 + \frac{1}{T}} = H_{\infty} DD_n$$
(2)

where  $T_n$  is the *null loop gain* and  $D_n = 1 + 1/T_n$  is the *null discrepancy factor*. Thus,  $H_{\infty}$ , T, and  $T_n$  are the three constituent transfer functions into which H is dissected, and are to be individually designed so that the overall specifications are met.

The value of the GFT is that the nonidealities are contained within the null loop gain  $T_n$ , and hence within the null discrepancy factor  $D_n$ . Thus the design problem acquires the additional step of ensuring that  $D_n$ , as well as D, is close enough to unity over the required frequency range. (Actually one nonideality, nonzero reverse loop gain, is also contained within the loop gain T, but the effect is usually negligible.)



Fig. 3. The GFT requires both voltage and current test signals to be injected at the feedback summing point.

Although the GFT can be employed in symbolic analysis, it is also computerfriendly. Intusoft's ICAP/4 design simulator includes GFT Templates, and also a User's Manual. The GFT Template quickly makes available Bode plots of all the simulated and post-processed transfer functions. A reduced Template produces the loop gain only, and reads out the phase and gain margins.

For a feedback amplifier with the structure of Fig. 3, for which H is a voltage gain and coupled voltage and current test signal injection is required, the appropriate GFT Template is GFTv chosen from the Parts Browser. It is connected to the SpiceNet circuit as shown in Fig. 4, so that  $v_y$  and  $i_y$  are respectively the error voltage and the error current, and both test signals  $e_z$  and  $j_z$  are injected inside the major loop but outside any minor loops that may exist inside the blocks. Note that both the input and output signals are connected to the Template icon, and the Template drive signal replaces the original input signal. Thus, all the sources and calculated signals are within the Template, and all the simulation runs and post-processing are controlled by the Template script.

In the Owner's Manual, how to set up and run an example of Fig. 4 is described in detail. The feedback ratio is a resistive divider  $K = R_1 / (R_1 + R_2) = 0.1$ , and the forward path is a two-stage discrete transistor amplifier.



Fig. 4. An appropriate GFT Template provides the injected test signals, and controls the simulation.



Fig. 5. Magnitude and phase of a loop gain T, showing a comfortable phase margin.

Results for the loop gain T, magnitude and phase, are shown in Fig. 5. The phase margin is a comfortable 87, so the discrepancy factor D in Fig. 6 does not exhibit any peaking. Also shown in Fig. 6 is the ideal closed-loop gain  $H_{\infty}$  and the product  $H_{\infty}D$ , which would be the final closed-loop gain if the nonidealities were negligible. Note that  $H_{\infty}$  is flat at 20dB, confirming that  $H_{\infty} = 1/K$ , the reciprocal of the feedback divider ratio.

However, in this example the nonidealities are not negligible, and the null discrepancy factor  $D_n$  deviates drastically from unity at higher frequencies as shown in Fig. 7. Consequently, the final closed-loop gain  $H = H_{\infty}DD_n$  also deviates drastically from  $H_{\infty}D$  at higher frequencies and exhibits what appears to be a large peak. Nevertheless, this is indeed the correct result for H as confirmed by a direct simulation without any injected test signals.

In the conventional approach, in which nonidealities are not recognized, the apparent peaking in the closed-loop gain might incorrectly be attributed to an inadequate phase margin. Moreover, this circuit has no ideal injection point, so use of only one injected test signal gives a drastically wrong result for the loop gain.



# Fig. 6. Ideal closed-loop gain $H_{\infty}$ and discrepancy factor D, whose product $H_{\infty}D$ would be the actual closed-loop gain in the absence of nonidealities.

The GFT of Eq. (2) defines a "natural" block diagram model that is identical in format to the single-loop model that is conventionally assumed, but with an added block  $H_0$  that represents the nonidealities, as in Fig. 8. There is a redundancy relation between  $H_0$  and  $T_n$ ,  $H_0/H_{\infty} = T/T_n$ , so that the nonidealities can be represented either by  $H_0$  or by  $T_n$  as may be convenient. The natural block diagram emerges as a *result* of the analysis, not as an initial assumption, and it is important to remember that, while the blocks in Fig. 8 are unidirectional, their transfer functions are calculated from the *entire* actual circuit including the effects of reverse transmission. Do not try to calculate the individual block transfer functions from what seems to be only the relevant part of the circuit!



Fig. 7. The null discrepancy factor  $D_n$ , containing the nonidealities, multiplies  $H_{\infty}D$  to give the actual closed-loop gain H.



Fig. 8. A "natural block diagram" emerges as a *result* of the GFT analysis, not as an initial assumption.

The GFT applies to any transfer function, so H can represent not only voltage and current gains, but also transfer gains and input and output impedances. There is a different GFT Template for each type of gain, and within each type there are additional Templates for different injected test signal combinations. The GFT can also represent the Extra Element Theorem (EET) to spotlight exactly how one or two selected elements affect the system performance. The GFT can be applied to any system able to be represented by a linear model, such as switching power converters, and is especially useful in analog high-frequency ICs where the nonidealities eventually control the performance.

The GFT Owner's Manual, available in the Help Files, contains further information on the GFT itself, and on a number of variations. It also contains a detailed walk-through of how to set up and solve the example circuit in Fig. 4 above, plus the GFT applied to the output impedance of the same circuit.

Still more information on the GFT, including several examples, are in a CD ROM "The GFT: A Final Solution for Design-Oriented Feedback Analysis" by Dr. R. David Middlebrook. Details of this CD ROM, and of Dr. Middlebrook's DVD ROM "Technical Therapy for Analog Circuit Designers," can be found at <u>http://www.rdmiddlebrook.com</u>.